

# What is effective teaching of mathematics?

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The old paradigm of balanced instruction focused on enabling children and teachers to succeed at school. Today, the focus is for students to achieve college and career readiness in life beyond school. This new paradigm expects that students:

- Construct their own understandings
- Apply prior knowledge and skills
- Are consistently challenged
- Take risks
- See purpose in what they learn

The goal is for students to be literate in mathematics so that we can prepare them for a world where the subject is rapidly growing and is extensively applied to a diverse number of fields.

Teaching mathematics can only be described as truly effective when it positively impacts student learning. We know that teaching practices can make a major difference to student outcomes, as well as what makes a difference in the classroom.

Research and evidence from the field of mathematics lets us know, with a fair degree of certitude, how effective teachers of mathematics skillfully integrate a range of instructional approaches and resources to meet the diverse learning needs of their students.

## Dimensions of Effective Mathematic Practice

### Effective Teachers:

- Know how students learn
- Know what students need to learn
- Know what their students already know
- Encourage risk taking
- Create purposeful learning experiences
- Create challenge

### Effective teachers:

- Know the pedagogy that determines how their students successfully learn
- Know and understand the content and practices that students need to comprehend, as described in the Standards framework
- Know the students they teach as learners
- Challenge all students at their own level
- Encourage risk taking
- Create purposeful learning experiences for students through the use of relevant and meaningful contexts

### Effective teachers know how students learn

Effective teachers of mathematics know the pedagogy that determines how their students successfully learn. Such teachers recognize that in order for students to effectively use mathematics they need to understand the concepts presented as well as become fluent with the skill taught. It is through the ongoing and increasingly complex application of concepts and skills that students become secure and competent in their use.

Effective teachers of mathematics are knowledgeable in the theory of learning their subject.

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Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. It means that both the curriculum and instruction should begin with problems, dilemmas, and questions for students.

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Hibert et al., 1996

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... our best evidence... is that what happens in classrooms through quality teaching and through the quality of the learning environment generated by the teacher and the students, is the key variable in explaining up to 59%, or even more, of the variance in student scores.

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Ministry of Education, 2003

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They recognize the importance of using concrete materials and visual representations to develop a deep understanding of the subject. They have a clear picture of the learning progression that best develops the knowledge base and skills of their students. They also have a broad palate of learning experiences they can use in the classroom, to meet the different learning needs of each student.

Effective teachers are able to look at student misconceptions, either in the classwork, through homework, or through assessments, and reteach the material using their understanding of the developmental nature of what becomes before or after the misconception. Deep understanding of the content enables teachers to directly address the specific misunderstandings that students may have.

Such teachers need to be continual learners. Effective pedagogy is the subject of ongoing research and development, and the way to teach and learn mathematics is never static.

#### Effective teachers know what students need to know

Effective teachers know and understand the content and practices of the mathematics Standards framework that students need to know. Such teachers have deep understanding of concepts and utilize multiple ways to represent and explain them. They are also fluent with the procedures and practices their students will need in order to succeed in mathematics.

The Common Core focus on career and college readiness requires that students be able to apply mathematics to complex problems in multiple contexts, both real and mathematical. As a consequence, this is also true for their teachers.

Effective teachers augment fluency procedure with:

- Deep conceptual understanding

- Knowledge of where and how to apply and use mathematics skills and concepts

Students need to use the above both in school, and in real world contexts.

#### Effective teachers know their students as learners

Knowing a student as a mathematics learner is complex. An effective mathematics teacher quickly builds a picture of their students by progressively providing opportunities to demonstrate what he/she is learning. This way, teachers update and deepen their understanding of individual students.

The effective teacher continuously uses this growing knowledge of students as learners to inform their instruction so they can better meet students' needs.

Assessment in mathematics is primarily formative. It involves collecting information from a range of sources, in a variety of ways. This includes information on students' strategies, understandings, attitudes, and prior knowledge and skills. Assessing a student involves making informed judgments about what the student knows. Hence, effective teachers not only monitor the performance of a student, but also their ability to show their understanding of the content that has been taught. Effective teachers:

- Integrate assessment into instructional practice
- Acknowledge students' *prior learning* and help them make *connections between what they already know and what they are currently learning*
- Gather information from a range of formal and informal sources using a variety of means, in particular written and verbal, and analyze the information presented
- Use *ongoing assessments to identify the learning needs* of each student. This allows them to teach pro-actively, assisting students to meet articulated goals

### Effective teachers create challenge

Each student learns best within their 'Zone of Proximal Development.' The effective teacher:

- Is able to identify and keep track of a student's 'Zone' through ongoing formative assessment
- Designs instruction that enables each student to learn within their 'Zone'
- Provides each student with challenges that meet their own level through the careful use of investigative tasks

When referring to "challenges," it is meant that a student will need to experience some degree of struggle to achieve a learning goal. An effective teacher will challenge every student to consistently operate at the upper end of their 'Zone of Proximal Development.'

Many strategies introduce challenges into a lesson, including the use of a rigorous open task, the use of questioning strategies (such as turn and talk) that involve all students, asking exploratory and generative questions, and consistently requiring students to pose, reflect on, and justify arguments.

Along with providing a challenge is the need to provide differing degrees of support for students. The greater the challenge, the greater the need for teacher support in a gradual release of responsibility from the teacher to the student.

### Effective teachers encourage risk taking

An optimal learning environment is one that generates an atmosphere of trust, where it is understood that making mistakes is acceptable, because mistakes are the essence of learning. A learning environment that promotes student mathematics learning is one where multiple strategies are encouraged. Understanding and recognizing connections within mathematics comes by finding different ways to achieve the same result.

The social setting that teachers provide is equally as important as the physical environment. Research tells us that the quality of the instruction makes the difference, and it is the interaction between the teacher and student, most especially the feedback the student gets, which is essential (Hattie, 2003).

Learning mathematics involves taking risks. This is more likely to happen if students feel that their attempts will be valued. Teachers need to help students see that mistakes are a natural part of the learning process. By accepting their approximations and providing informed, genuine, and encouraging responses, teachers are letting students know that they believe in them. While teachers can create the conditions that foster learning, and demonstrate strategies for learning to occur, the students have to actively be engaged. The program needs to be interesting enough so that students will want to engage in the learning.

### Effective teachers create purposeful learning experiences

Effective teachers of mathematics create purposeful learning experiences for students through solving problems in relevant and meaningful contexts.

There is ample evidence showing the need for problem-solving to be an integral part of all mathematics learning. Teaching through problem solving, however, means that students learn mathematics through real contexts, problems, situations, and models. The contexts and models allow students to build meaning for the concepts. This way, they can move on to increasingly more abstract concepts.

A problem can be defined as

*'... any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific "correct" solution method'* - Hiebert et al., 1997

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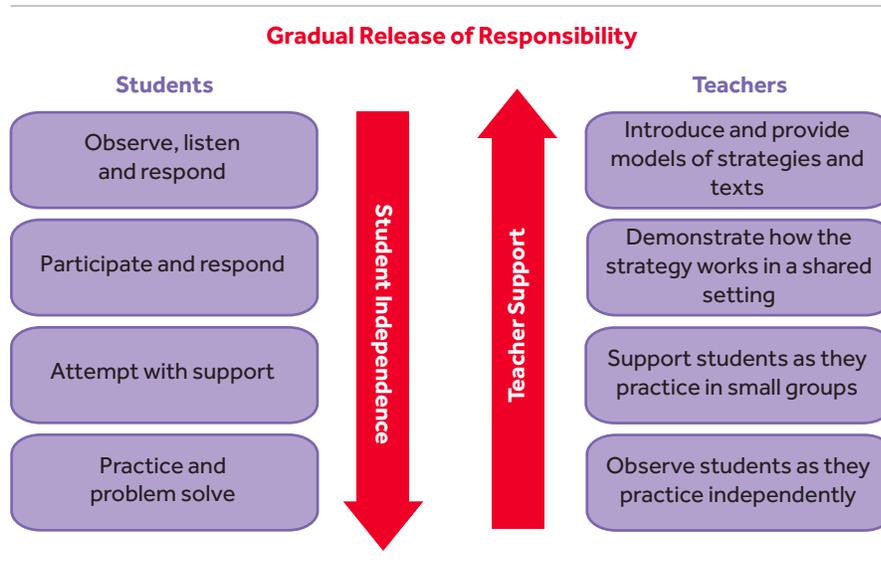
... Learning mathematics with understanding is essential. Mathematics today requires not only computational skills but also the ability to think and reason mathematically in order to solve the new problems and learn the new ideas that students will face in the future.

... Learning is enhanced in classrooms where students are required to evaluate their own ideas and those of others, are encouraged to make mathematical conjectures and test them, and are helped to develop their reasoning skills.

John Van De Walle

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Effective teachers will use such problems as starting points and an ongoing means for students to investigate and understand conceptual ideas so they can develop skills and procedures. Through such problems, all students are provided with appropriate entry points to progressively develop the understanding of concepts and increasingly more complex skills that facilitate efficient problem solving.



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